

Two-parton contribution to the heavy-quark A_{FB} at NNLO QCD in e^+e^- collisions

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At the next generation of linear colliders, forward-backward asymmetries will be involved again in the precise determination of the neutral current couplings of heavy quarks in inclusive heavy-quark production. The theoretical understanding of these observables at the level of NNLO radiative corrections is required. We review a recent calculation of the two-parton contribution to the heavy-quark A_{FB} at NNLO QCD in e^+e^- collisions. The results are valid for arbitrary values of the momentum transfer and non-vanishing heavy-quark mass m_Q .

The precision measurements of the forward-backward asymmetries in the production of fermions at high-energy e^+e^- colliders provide the determination of the respective fermion neutral current couplings with a remarkable accuracy. For instance, the forward-backward asymmetry of b quarks measured at the Z resonance with a 1.7% accuracy, led to a determination of $\sin^2 \theta_{W,eff}$ of the Standard Model with a relative precision of about 1 per mille [1,2].

At a future linear e^+e^- collider [3], precision determinations of electroweak parameters will again involve forward-backward asymmetries. When such a collider will be operated at the Z peak, accuracies of about 0.1% may be reached for these observables [4,5]. Moreover, the top quark asymmetry A_{FB}^t will be experimentally accessible. Therefore, it becomes important a theoretical understanding of these observables at the level of NNLO radiative corrections.

At present, the forward-backward asymmetry for the c and b quarks are known at the level of NLO electroweak [6,7,8] and fully massive NLO QCD [9,10,11] corrections. The NNLO QCD corrections were calculated in the limit of massless quarks in [12,13] and retaining logarithmically-enhanced terms of the type $\ln(Q/m_Q)$ in [14], where the calculation of A_{FB}^b is done both with

respect to the quark and the thrust axis.

In view of the future perspectives for the b - and t -quark asymmetries at a linear collider, a computation of the order α_s^2 contributions to A_{FB}^Q for massive quarks Q is clearly desirable. The NNLO QCD corrections involve three classes of contributions: (1) the two-loop corrections to the decay of a vector boson into a heavy quark-antiquark pair; (2) the one-loop corrected matrix elements for the decay of a vector boson into a heavy quark-antiquark pair plus a gluon; (3) the tree level matrix elements for the decay of a vector boson into four partons, at least two of which being the heavy quark-antiquark pair.

The contributions (1) from the two-parton final state, and (2) plus (3), i.e., those from the three- and four-parton final states, are separately infrared-finite. The latter can be obtained along the lines of the calculations of three-jet production involving heavy quarks [15,16,17,18]. However, a full computation of $A_{FB}^{3+4\text{parton}}$ has not yet been done for massive quarks.

For what concerns the contribution (1), instead, an analytic expression valid for non-vanishing heavy-quark mass m_Q and arbitrary momentum transfer was given in [19]. This expression lies on a previous calculation of the NNLO QCD corrections to the form factors for the vertices γQQ and ZQQ [20,21,22,23], done using the Laporta algorithm [24] for the reduction of the dimensionally-regularized scalar integrals

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to the set of master integrals and the differential equations technique [25] for their calculation [26] in terms of harmonic polylogarithms [27] (all the calculations are done in FORM [28]).

Forward-backward asymmetry for heavy quarks can be defined as the ratio of the “antisymmetric” and “symmetric” cross sections, σ_A and $\sigma_S = \sigma$,

$$A_{FB} = \frac{\sigma_A}{\sigma_S}, \quad (1)$$

where σ_A and σ_S are defined by:

$$\sigma_A = \int_0^1 \frac{d\sigma}{d\cos\vartheta} d\cos\vartheta - \int_{-1}^0 \frac{d\sigma}{d\cos\vartheta} d\cos\vartheta, \quad (2)$$

$$\sigma_S = \int_{-1}^1 \frac{d\sigma}{d\cos\vartheta} d\cos\vartheta. \quad (3)$$

ϑ is the angle between the incoming electron and the direction defining the forward hemisphere (in the e^+e^- center-of-mass frame), that must be infrared- and collinear-safe in order that A_{FB} is computable in perturbation theory. Common choices are the direction of flight of Q or the thrust axis direction. In the case of the $Q\bar{Q}$ contribution to A_{FB} , $A_{FB}^{(2p)}$ (two-parton contribution), the result in the two cases is the same.

Expanding in series of α_s both σ_A and σ_S , and retaining only the contributions coming from the two-parton final state, we can write $A_{FB}^{(2p)}$ as follows:

$$A_{FB}^{(2p)} = A_{FB,0} \left[1 + A_1^{(2p)} + A_2^{(2p)} \right], \quad (4)$$

where $A_{FB,0}$ is the forward-backward asymmetry at the Born level, and $A_1^{(2p)}$ and $A_2^{(2p)}$ are the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ contributions normalized to $A_{FB,0}$. Their expressions in terms of the form factors can be found in [19]. Note that, although the single form factors, ultraviolet-renormalized, contain still infrared divergencies, the two-parton contributions to the forward-backward asymmetry $A_1^{(2p)}$ and $A_2^{(2p)}$ are infrared-finite. This is due to the factorized structure of the infrared singularities at the level of the form factors. For massless multiloop amplitudes the factorization of the infrared poles can be derived from exponentiation

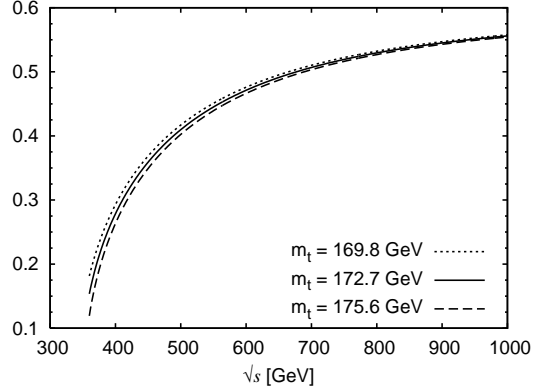


Figure 1. Leading order asymmetry $A_{FB,0}^{(t\bar{t})}$ for three values of the top quark mass.

[29,30]. In the case of massive QCD, infrared factorization was established at the one-loop level in [31], and the pole structure of the massive form factors calculated in [20,21,22,23] suggests that it holds at higher orders as well.

In Fig. 1 $A_{FB,0}$ is shown in the case of production of a $t\bar{t}$ pair, for three different values of the mass of the top quark and for c.m. energy up to 1 TeV. In Fig. 2 we give the order α_s correction, $A_1^{(2p)}$, for three different values of the renormalization scale μ . This correction ranges between 4% near the threshold of production of the $t\bar{t}$ pair and less than 1% at 1 TeV. At the NNLO in QCD, $A_2^{(2p)}$ takes two kind of contributions called type A and type B contributions. Type A contributions are those where the Z boson couples directly to the external quark Q [21], while the triangle diagram contributions, summed over the quark isodoublets of the three generations, are called type B [22]. $A_2^{(t\bar{t},A)}$ and $A_2^{(t\bar{t},B)}$ are shown in Fig. 3, for three values of the renormalization scale. Finally, in Fig. 4 it is shown the forward-backward asymmetry to lowest, first and second order in α_s , always in the case of top-quark production, for $m_t = 172.7$ GeV and $\mu = \sqrt{s}$.

In general the QCD corrections increase as far as the c.m. energy approaches the threshold region, signaling that perturbation theory in α_s is no longer applicable. At $\sqrt{s} = 400$ GeV, $A_1^{(t\bar{t})}$ is about 3.3% while $A_2^{(t\bar{t})}$ is about 2.4%. Increasing

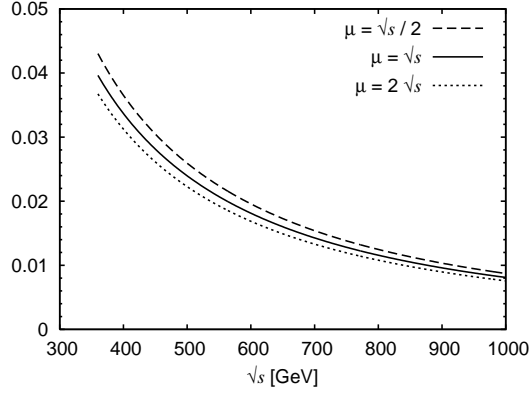


Figure 2. Order α_s correction $A_1^{(t\bar{t})}$ for three values of the renormalization scale μ , using $m_t = 172.7$ GeV.

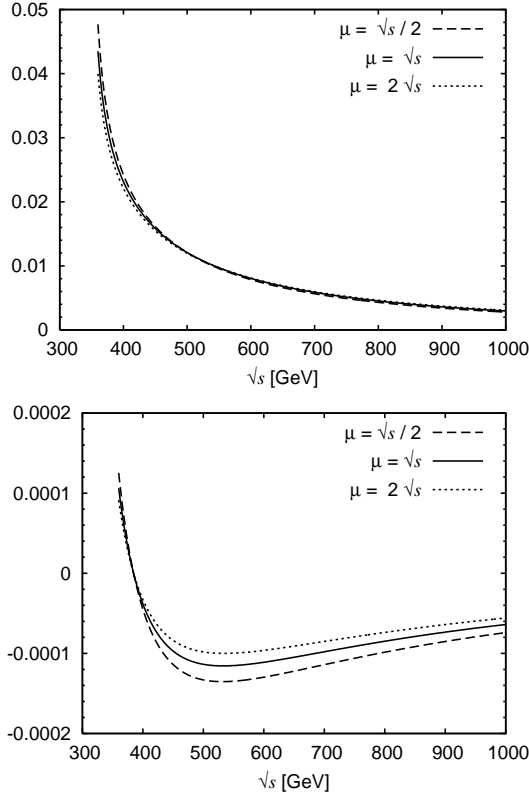


Figure 3. Order α_s^2 correction $A_2^{(t\bar{t},A)}$ (upper figure) and $A_2^{(t\bar{t},B)}$ (lower figure) for three values of the renormalization scale μ , using $m_t = 172.7$ GeV.

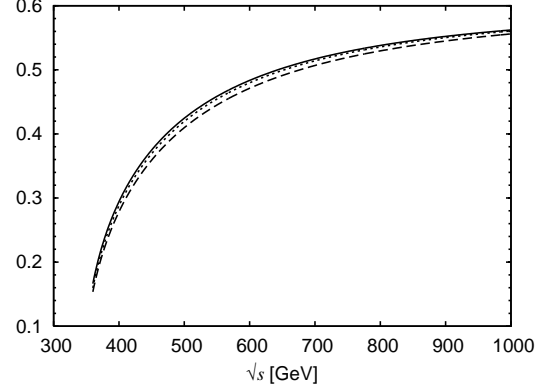


Figure 4. Forward-backward asymmetry to lowest, first and second order in α_s using $m_t = 172.7$ GeV and $\mu = \sqrt{s}$. $A_{FB,0}^{(t\bar{t})}$ (dashed), $A_{FB}^{(t\bar{t})}(\alpha_s)$ (dotted), $A_{FB}^{(t\bar{t})}(\alpha_s^2)$ (solid).

the c.m. energy, the relative importance of the radiative corrections decreases. Moreover, we see that the QCD corrections do not depend strongly on the change of the renormalization scale μ .

In [19,32], a detailed analysis of the behaviour of the exact formulas in the energy region near the threshold of production of the $t\bar{t}$ pair is also done. For values of the quark velocity β such that $\alpha_s \ll \beta \ll 1$, the heavy-quark production cross section can be written as follows:

$$\begin{aligned} \sigma_{NNLO} = \sigma_S^{(2,0,\gamma)} & \left\{ 1 + \Delta^{(0,Ax)} \right. \\ & + C_F \left(\frac{\alpha_s}{2\pi} \right) \Delta^{(1,Ve)} (1 + \Delta^{(0,Ax)}) \\ & + C_F \left(\frac{\alpha_s}{2\pi} \right) \Delta^{(1,Ax)} \\ & + C_F \left(\frac{\alpha_s}{2\pi} \right)^2 \Delta^{(2,Ve)} (1 + \Delta^{(0,Ax)}) \\ & \left. + C_F \left(\frac{\alpha_s}{2\pi} \right)^2 \Delta^{(2,Ax)} \right\}, \end{aligned} \quad (5)$$

where the various terms are given in Laurent series of β (Coulomb poles in $1/\beta$ are present). At order β^0 the cross section in Eq. (5) is infrared finite. The analytic expressions of $\Delta^{(i,Ve)}$ and $\Delta^{(0,Ax)}$ agree with the results in [33,34,35,36,37], while in [19,32] the term

$$\Delta^{(2,Ax)} = \frac{64\zeta(2)m_Q^4(a_Q^Z)^2[(v_e^Z)^2 + (a_e^Z)^2]}{(v_Q^\gamma v_e^\gamma)^2(4m_Q^2 - m_Z^2)^2} C_F \quad (6)$$

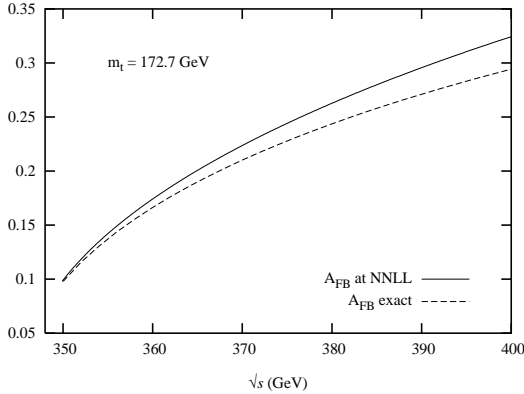


Figure 5. The second order forward-backward asymmetry $A_{FB}^{(t\bar{t})}(\alpha_s^2)$ near threshold: exact values (dashed) as given in Fig. 4 and the values obtained from the near-threshold formula (solid), using $\mu = m_t = 172.7$ GeV.

is also given. These results, however, can not be used to extract the matching coefficients at two loops between QCD and NRQCD [38].

Using Eq. (5) and an analogous expression for the antisymmetric cross section, we can compute the forward-backward asymmetry near the threshold, that, at this order in β , is equal to the complete forward-backward asymmetry $A_{FB}^{(t\bar{t})}$. In Fig. 5 we show the comparison between the exact second order forward-backward asymmetry $A_{FB}^{(t\bar{t})}(\alpha_s^2)$ as given in Fig. 4 and the values obtained from the near-threshold formula. For $\sqrt{s} \lesssim 360$ GeV corresponding to $\beta \lesssim 0.3$ the deviation of the threshold from the respective exact value is less than 5%.

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